系所：　數學系　　　　　　　　　　　　　年級：　二　　　

科目：　微積分　　　　　　　　　

☆☆請在答案紙上作答☆☆　　　　　　　　　　　　　　　　共 1 頁，第 1 頁

1. Prove, by $\varepsilon - \delta$ limit definition, $\lim\limits_{x \to 2} \dfrac{1}{\sqrt{x}} = \dfrac{1}{\sqrt{2}}$.(10%)

2. (a) Show that $f(x) = x\sqrt{x^4 + 2}$ is one to one on $(-\infty, \infty)$.(5%)

(b) Find $(f^{-1})'(\sqrt{3})$. (5%)

3. Evaluate the following limits: You need to show your processes of getting answers.(30%)

(a) $\lim\limits_{x \to \infty} \sqrt{4x^2 + 2x} - 2x$ (b) $\lim\limits_{x \to 0} \dfrac{\sin x^2}{x}$ (c) $\lim\limits_{x \to 0} \dfrac{x - \sin x}{6x^3}$ (d) $\lim\limits_{x \to \infty} e^{-x}\sqrt{x}$ (e) $\lim\limits_{x \to \infty}(1 + \dfrac{1}{x})^x$.

(f) $\lim\limits_{x \to 0^+} (4x)^{\sqrt{x}}$

4. Find the following indefinite integrals :(20%)

(a) $\displaystyle\int \dfrac{\sqrt[3]{x}}{\sqrt[3]{x} - 1}dx$ (b) $\displaystyle\int e^{\sqrt{x}}dx$.

5. Determine whether the following series converges or diverges. Show your answer.

(1) $\displaystyle\sum_{n=1}^{\infty} n\left(\sin \dfrac{1}{n^4}\right)$ 　　　(5%) 　　　　(2) $\displaystyle\sum_{n=1}^{\infty} \dfrac{n^2}{\sqrt[3]{n} + n^3}$ 　　　(5%)

6. Evaluate $\displaystyle\int_0^2 (\int_y^2 e^{-x^2}dx\,)dy$. (10%)

7. Use Green's Theorem to evaluate the line integral (10%)

$$\int_C y^3\, dx + (x^3 + 3 x^2 y)\,dy$$

where C is the path from $(0,0)$ to $(1,1)$ along the graph of $y = x^3$ and from $(1,1)$ to $(0,0)$ along the graph of $y = x$.