國立彰化師範大學106學年度第1學期學士班轉學生招生考試試題

學系：數學系　　年級：二年級

科目：微積分

☆☆請在答案紙上作答☆☆　　

1. Evaluate the following limits.

(1) $\lim_{x\to 0}\dfrac{\sqrt{x^2+9}-3}{x^2}$ (5%)　　**(2)** $\lim_{x\to \frac{\pi}{2}^-}(\sec x-\tan x)$ (5%)　　**(3)** $\lim_{x\to 0^+}(2x)^x$ (5%)

2. Find the derivatives of following functions if it exists**.**

(1) $f(x)=\ln\sqrt{x^2+1}$ (5%)　　**(2)** $f(x)=\sin|\cos x|$ (5%)

(3) $f(x)=\int_{1-x}^{1+x}\ln(t^2+1)dt$ (5%)　　**(4)** $f(x)=x^{2x}$ (5%)

3. Evaluate $\lim_{n\to\infty}\sum_{i=1}^{n}\frac{3}{n}\left(1+\frac{3i-1}{n}\right)^2$**.** (10%)

4. Find the function y(x) satisfying the following conditions:

$\dfrac{dy}{dx}=\dfrac{3+2x^2}{x^2}$ **on** $(-\infty,0)$ **and** $y(-2)=1$**.** (10%)

5. Determine convergence or divergence of the following series and prove your answer.

(1) $\sum_{n=1}^{\infty}\ln\frac{1}{\sqrt{n}}$ (5%)　　**(2)** $\sum_{n=1}^{\infty}\sin\frac{1}{n^2}$ (5%)

6. Find the following limits if it exists; if the limit does not exist , explain why.

(1) $\lim_{(x,y,z)\to(0,0,0)}\dfrac{xy+yz+xz}{x^2+y^2+z^2}$ (7%)　　**(2)** $\lim_{(x,y)\to(0,0)}\dfrac{xy^2}{x^2+y^2}$ (8%)

7. Let $w=f(x,y)$ **be differentiable on** R^2**, and let** $x(u,v)=u-v$ **and** $y(u,v)=v-u$**. If** $w=f(x(u,v),y(u,v))$**, then show** $\dfrac{\partial w}{\partial u}+\dfrac{\partial w}{\partial v}=0$**.** (10%)

8. Find the volume of the solid that lies under the paraboloid $z=x^2+y^2$ **and above the region D in the xy-plane bounded by the line** $y=2x$ **and the parabola** $y=x^2$**.** (10%)