國立彰化師範大學106學年度第2學期學士班轉學生招生考試試題

系所：數學系　　年級：二

科目：微積分

☆☆請在答案紙上作答☆☆　　

1. Use the $\varepsilon-\delta$ definition to show that $\lim_{x\to 1} x^2 = 1$. (10 %)

2. Evaluate the following integrals:

   (1) $\int_1^2 \frac{2x+1}{x(x^2+1)}\,dx$. (10 %)

   (2) $\int_0^\infty e^{-x}\sin x\,dx$. (10 %)

3. Let $f:[-1,1]\to\mathbb{R}$ be a function defined by

$$f(x)=\begin{cases} x\ln|x| & ,x\neq 0 \\ 0 & ,x=0 \end{cases}.$$

   (1) Show that the function is continuous on $[-1,1]$, but not differentiable at $x=0$. (10 %)

   (2) Find the volume of the solid generated by revolving the region bounded by the graph of $y=f(x)$ and $y=0$, about the x-axis. (10 %)

4. Determine the set of all x that makes the following power series converge, and explain why:

   (1) $\sum_{n=1}^{\infty}\frac{(x-1)^n}{3^n\sqrt{n}}$ (10 %)

   (2) $\sum_{n=1}^{\infty}\frac{n!x^n}{n^n}$ (10 %)

5. Find the 3rd order Taylor polynomial at $x=0$ for $f(x)=e^{\frac{-x^2}{2}}\sin x$. (10 %)

6. Find the maximum and minimum of $f(x,y)=x^2+3xy+y^2$ on the domain

   $D=\{(x,y)\,|\,x^2+y^2\le 1\}$. (10 %)

7. Evaluate the integral $\iint_D xe^y dA$, where D is the triangle bounded by $x+y=4, x=0,$ and $y=0$. (10 %)