

國立彰化師範大學107學年度第2學期學士班轉學生招生考試試題

系所： 數學系

年級： 二

組別： 不分組

科目： 微積分

☆☆請在答案紙上作答☆☆

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1. (12%) Prove, by the ε - δ limit definition, $\lim_{x \rightarrow 2} \frac{1}{x} = \frac{1}{2}$.

2. (10%) Find all inflection points of the function $f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$.

3. (20%) Evaluate the following limits:

You need to show your processes of getting answers.

(a) $\lim_{x \rightarrow \infty} \left(1 - \frac{1}{x}\right)^x$ (b) $\lim_{x \rightarrow 0^+} x^{\sqrt{x}}$ (c) $\lim_{x \rightarrow 0} \frac{\tan(\sin^{-1} x)}{\sin(\tan^{-1}(3x))}$ (d) $\lim_{x \rightarrow 0} x \sin\left(\frac{2}{x}\right)$

4. (20%) Find the following indefinite integrals:

(a) $\int \left(x^e + e^x + \frac{x}{e} + \frac{e}{x} + e\right) dx$ (b) $\int \frac{1}{x\sqrt{\ln x}} dx$

(c) $\int e^{\sqrt{x}} dx$ (d) $\int \cos^2(3x) dx$

5. (10%) Determine whether the following series converges or diverges. Show your answer.

(a) $\sum_{n=1}^{\infty} n \sin\left(\frac{1}{n^3}\right)$ (b) $\sum_{n=1}^{\infty} \frac{n^2}{\sqrt[4]{2n} + n^3}$.

6. (10%) Evaluate the iterated integral $\int_0^2 \left(\int_y^2 e^{-x^2} dx\right) dy$.

7. (12%) Use Green's Theorem to evaluate the line integral

$$\int_C y^3 dx + (x^3 + 3xy^2) dy,$$

where C is the path from $(0,0)$ to $(1,1)$ along the graph of $y = x^3$ and from $(1,1)$ to $(0,0)$ along the graph of $y = x$.

8. (6%) Find the directional derivative of $f(x,y) = e^{xy^2}$ at $(3,-1)$ in the direction

$$v = \left(\frac{3}{5}, \frac{4}{5}\right).$$