(20%)　　1. Determine whether the linear transformation T : $R^3 \to R^3$ defined by

$$T([x, y, z]) = [x - y + 3z, x + y + z, x]$$

　　　is invertible.　　If yes, find a formula for its inverse.

　　　2. Let V and V′ be vector spaces with ordered bases $B = (1, x, x^2 - x)$ and

　　$B' = (1, \cos x, \sin x)$ respectively.　Let $T : V \to V'$ be the linear transformation such that

　　　　$T(1) = 1 + 2\cos x - 3\sin x$,

　　　　$T(x) = 3 + 5\cos x + 2\sin x$,

　　　　$T(x^2 - x) = -2 - 3\cos x - 4\sin x$.

(10%)　　(a) Find the matrix representation A of T relative to the ordered bases B and B'.

(10%)　　(b) Use A to find $T(v)_{B'}$ if $v = 2 - 5x + x^2$.

(10%)　　(c) Show that T is invertible and find the matrix representation of T^{-1} relative to B' and B.

(20%)　　3. Find an orthonormal basis for the subspace

　　$W = \{[x_1, x_2, x_3, x_4] \in R^4 \mid 2x_1 - 3x_3 + x_4 = 0, 3x_1 + 4x_2 + 2x_3 + 2x_4 = 0\}.$

(20%)　　4. Compute A^{50}, where A is a matrix given by

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}.$$

(10%)　　5. Show that $sp(v_1, v_2) = sp(2v_1 + v_2, v_1 - 3v_2)$, where $v_1, v_2 \in R^3$ and

　　$sp(v_1, v_2)$ denotes　the set of all linear combinations of v_1 and v_2.

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