國立彰化師範大學106學年度第1學期學士班轉學生招生考試試題

學系：　數學系　　　　　　　　　　　　　　　　　年級：　二年級

科目：　線性代數

☆☆請在答案紙上作答☆☆　　　　　　　　　　　　　　

1. Let $A=\begin{bmatrix} 2 & 3 \\ -2 & -6 \\ 1 & 0 \end{bmatrix}$ Find $\vec{x}=\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ such that $\left\|A\vec{x}-\vec{b}\right\|$ attains a minimum,

   where $\vec{b}=\begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$.(15%)

2. Find the solution set *W* for the linear system.

$$\begin{cases} x_1 \qquad\quad -2x_3+x_4=3 \\ 2x_1-\ \ x_2+x_3-3x_4=0 \\ 9x_1-3x_2-x_3-7x_4=4 \end{cases}$$.(20%)

3. Find an orthogonal 2×2 matrix Q such that $Q\vec{x}=\vec{w}$,

   where $\vec{x}=\begin{bmatrix} 7 \\ 24 \end{bmatrix}$ and $\vec{w}=\begin{bmatrix} -25 \\ 0 \end{bmatrix}$. (15%)

4. Let $T: P_2 \to P_2$ be defined by $T(p(x)) = p'(x) + p(x+1)$,

where P_2 is the set of all polynomials of degree ≤ 2 .

Find the matrix representations R_B and $R_{B'}$ and an invertible matix

C such that $C^{-1}R_BC = R_{B'}$ for the linear tramsformation T relative

to the ordered bases $B = (1,\ x^2,\ x)$ and $B' = (x-1,\ x^2+1,\ 2)$.(20%)

5. Find the value of k that satisfies the following equation

$$\det\begin{bmatrix} 2a_1 & 2a_2 & 2a_3 \\ 3b_1+5c_1 & 3b_2+5c_2 & 3b_3+5c_3 \\ 7c_1 & 7c_2 & 7c_3 \end{bmatrix} = k\det\begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$$.(15%)

6. Let $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

Find an orthogonal matrix C such that $C^{-1}AC$ is an diagonalization of A. (15%)