1. Find the solution set of the following linear system.　(16%)

$$x_1 - 2x_2 + x_3 + x_4 = 4$$
$$2x_1 + x_2 - 3x_3 - x_4 = 6$$
$$x_1 - 7x_2 - 6x_3 + 2x_4 = 6$$

2. Prove that the set $\{\sin(x),\ \cos(x),\ \sin(2x),\ \cos(2x)\}$ is linearly independent.　(16%)

3. Let $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

   Find an orthogonal matrix C such that $C^{-1}AC$ is a diagonal matrix.　(16%)

4. Let T be a linear transformation from $\mathbb{R}^2$ into $\mathbb{R}^2$ given by $T(b_1) = -b_1$, and $T(b_2) = b_2$,

   where $b_1 = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ and $b_2 = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$

   Find an 2×2 matrix A such that $T(x) = Ax$ $\forall x \in \mathbb{R}^2$　(16%)

5. Let V be an inner-product space, and that v and w be vectors in V.

   Show that $|<v,\ w>| \le \|v\|\ \|w\|$.　(16%)

6. Use the Gram-Schmidt process to find an orthogonal basis $\{v_1,\ v_2,\ v_3\}$ for the subspace

   $\mathrm{sp}(1,\ x,\ x^2)$ of the vector space $C_{0,2}$ of continuous functions with domain $0 \le x \le 2,$

   where $<f,g> = \int_0^2 f(x)g(x)dx$.　(20%)