國立彰化師範大學107學年度第2學期學士班轉學生招生考試試題

系所:<u>數學系</u>年級:<u>二</u>組別:<u>不分組</u>

科目: 線性代數

☆☆請在答案紙上作答☆☆

共1頁,第1頁

1. (20%) Compute the rank and the inverse of H, if it exists, where

$$H = \begin{pmatrix} 1 & 2 & 1 \\ -1 & 1 & 2 \\ 1 & 0 & 1 \end{pmatrix}.$$

2. (20 %) Express the invertible matrix J as a product of elementary matrices, where

$$J = \begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix}.$$

3. (20%) Find the solution set of the linear system of equations

$$\begin{cases} x_1 + 2x_2 + x_3 + 4x_4 = 1 \\ 2x_1 - x_2 - 3x_3 + 3x_4 = 4 \\ 3x_1 + 7x_2 + 5x_3 + 11x_4 = 8 \end{cases}.$$

4. (20%) Let

$$A = \begin{pmatrix} 3 & 1 & 1 \\ 2 & 4 & 2 \\ -1 & -1 & 1 \end{pmatrix} .$$

- (1) Find the eigenvalues of A.
- (2) Find an invertible matrix C and a diagonal matrix D such that $C^{-1}AC = D$.
- 5. (20%) Let

$$B = \begin{pmatrix} 3 & x & 1 \\ x & 1 & 2 \\ -1 & -1 & 1 \end{pmatrix} .$$

Find all real numbers x such that the matrix B is not invertible.