

# 國立彰化師範大學115學年度碩士班招生考試試題

系所： 數學系(選考丙)、  
統計資訊研究所(選考丙)

科目： 微積分

☆☆請在答案紙上作答☆☆

共1頁，第1頁

1. (10 points) Let  $g(x) = \sin(\sqrt{3x^2 + \sec^2(2x)})$ . Evaluate the derivative  $g'(0)$ .

2. (10 points) Define

$$f(x) = \begin{cases} x^4, & x < 0, \\ 0, & x = 0, \\ x^2, & x > 0. \end{cases}$$

Determine whether  $f$  is differentiable at 0. If it is, compute  $f'(0)$  and prove your claim.

3. (a) (10 points) State the Mean Value Theorem.

(b) (10 points) Use the Mean Value Theorem to prove that

$$\ln(1+x) < x,$$

for all  $x > 0$ .

4. (10 points) Suppose  $g(x, y) = (2x - 1)y$ ,  $D = \{(x, y) \in \mathbb{R}^2: x^2 + y^2 \leq 4\}$ . Find the maximum and minimum of  $g$  on  $D$ .

5. (10 points) Evaluate the definite integral

$$\int_0^{\pi/4} \tan^4 x \, dx.$$

6. (15 points) Find the Maclaurin series for  $f(x) = \sin x$  and show that the series converges to  $f$  for all  $x \in \mathbb{R}$ .

7. (10 points) Find the volume of the solid region bounded by the surface  $f(x, y) = e^{-x^2}$  and the planes  $z = 0$ ,  $y = 0$ ,  $y = x$ , and  $x = 1$ .

8. (15 points) Let  $f: \mathbb{R}^2 \rightarrow \mathbb{R}$  be a function and  $L \in \mathbb{R}$ . Determine whether the following statement is true or false: If  $\lim_{x \rightarrow 0} f(x, 0) = \lim_{y \rightarrow 0} f(0, y) = L$ , then  $f$  is continuous at  $(0, 0)$ . Justify your answer with a proof or a counterexample.