

國立彰化師範大學115學年度碩士班招生考試試題

系所：數學系(選考甲)、
統計資訊研究所(選考甲)

科目：線性代數

☆☆請在答案紙上作答☆☆

共1頁，第1頁

1. (20%) Find the solution set of the system of linear equations :

$$\begin{cases} x_1 + 2x_2 + 2x_3 - 3x_4 = 4 \\ 3x_1 + x_2 - 2x_3 + x_4 = 3 \\ x_2 + x_3 - 2x_4 = 0 \end{cases}$$

2. (20%) Let $A = \begin{bmatrix} 2 & 5 \\ 5 & 2 \end{bmatrix} \in M_{2 \times 2}(\mathbb{R})$ be a 2×2 real matrix.

Find an invertible matrix P and a diagonal matrix D such that $P^{-1}AP = D$.

3. Let \mathbb{R}^4 be a real inner product space with the standard inner product.

Let $W = \text{span}(\{[1, -1, 0, 0], [1, 0, 1, 0], [0, 1, 1, 1]\})$ be a subspace of \mathbb{R}^4

and let W^\perp be the orthogonal complement of W in \mathbb{R}^4 .

- (1) (15%) Find an orthogonal basis for W .

- (2) (10%) Find an orthogonal basis for W^\perp .

4. (1) (10%) Let $A = \begin{bmatrix} 1 & x_1 & x_1^2 & x_1^3 & x_1^4 \\ 1 & x_2 & x_2^2 & x_2^3 & x_2^4 \\ 1 & x_3 & x_3^2 & x_3^3 & x_3^4 \\ 1 & x_4 & x_4^2 & x_4^3 & x_4^4 \\ 1 & x_5 & x_5^2 & x_5^3 & x_5^4 \end{bmatrix} \in M_{5 \times 5}(\mathbb{R})$ be a 5×5 real matrix.

Prove that $\det(A) = \prod_{1 \leq i < j \leq 5} (x_j - x_i)$.

- (2) (10%) Suppose that $B = \begin{bmatrix} 1 & 2 & 4 & 8 & 16 \\ 1 & 3 & 9 & 27 & 81 \\ 1 & 4 & 16 & 64 & 256 \\ 1 & 5 & 25 & 125 & 625 \\ 1 & s & s^2 & s^3 & s^4 \end{bmatrix} \in M_{5 \times 5}(\mathbb{R})$ is a 5×5 real matrix.

Find all $s \in \mathbb{R}$ such that B is an invertible matrix.

5. Let $A \in M_{2 \times 2}(\mathbb{R})$ be a 2×2 real matrix.

Suppose that $A^n = O$ for some positive integer n , where O denotes the zero matrix.

- (1) (5%) Find all eigenvalues of A .

- (2) (10%) Prove that $A^2 = O$.