國立彰化師範大學105學年度碩士班招生考試試題

系所：數學系　　組別：乙組　　科目：微積分

☆☆請在答案紙上作答☆☆　　

1. Show that $f(x)=x\sqrt{x^2+3}$ is one to one on $(-\infty,\infty)$. Find $(f^{-1})'(2)$. (10%)

2. Evaluate the following limits

(1) $\lim_{x->\infty}\sqrt{x^2+x}-x$ (5%)　(2) $\lim_{x->2^-}\frac{|x-2|}{x^2+x-6}$ (5%)　(3) $\lim_{x->0+}(5x)^{\sqrt{x}}$ (10%)

3. Evaluate the following integrals

(1) $\int x\ \sin^{-1}(2x)\ dx$ (5%)　(2) $\int\frac{3+2x^2}{x^2}\ dx$ (5%)

(3) $\int\frac{1}{x}\ dx$ (5%)　(4) $\int\frac{x+1}{(x^2+1)^2}\ dx$ (5%)

4. Determine whether the following series converges or diverges by using any appropriate test

(1) $\sum_{n=1}^{\infty}\sin\frac{1}{n^3}$ (5%)　(2) $\sum_{n=1}^{\infty}\frac{1}{\sqrt[3]{n}+n^3}$ (5%)

5. Evaluate $\int_0^1(\int_y^1 e^{-x^2}dx\ )dy$. (10%)

6. Evaluate the line integral $\int_C(x^2+3y)\ ds$ over the given curve C:

$r(t)=(\ t\ ,\ 2t\ +1\ ,\ 3t\ +2\ ),\ 0\le t\le 1.$ (10%)

7. Let $f(x)=x\sin\frac{1}{x}$ for $x\neq 0$ and $f(x)=0$ for $x=0$. Does $f'(0)$ exist? Please explain your answer. (10%)

8. Evaluate $\int_0^4\int_{\frac{y}{2}}^{\frac{y}{2}+1}\frac{2x-y}{2}dx\ dy$. (10%)