

國立彰化師範大學106學年度碩士班招生考試試題

系所： 數學系

組別： 乙組

科目： 微積分

☆☆請在答案紙上作答☆☆

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1. Let $f(x) = \begin{cases} ax^3, & x \leq 2 \\ x^2 + b, & x > 2 \end{cases}$. Find a and b such that f is differentiable at 2. (20%)

2. Let $f(x) = \begin{cases} x^2 \sin \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$. Prove that f is differentiable at 0. (20%)

3. For $f(x) = x^3 - \frac{3}{2}x^2$, prove that f is increasing on the open interval $(-3, 0)$. (20%)

4. If f is continuous on $[a, b]$ and differentiable on (a, b) such that $f(b) = f(a)$, prove that there is an $c \in (a, b)$ such that $f'(c) = 0$. (20%)

5. Prove that $\sum_{n=0}^{\infty} \frac{\sqrt{n}}{n^2+1}$ converges. (20%)