系所：　數學系(選考丙)、　　　　　　　　　　　　科目：　微積分
　　　　統計資訊研究所(選考丙)

☆☆請在答案紙上作答☆☆　　　　　　　　　　　　　　　　共 1 頁，第 1 頁

1. Show by using the $\epsilon - \delta$ definition that $\lim\limits_{x \to 1} \dfrac{1}{x} = 1$. (15%)

2. Evaluate the double integral $\displaystyle\int_0^{\ln 5} \int_{e^x}^5 \dfrac{1}{\ln y}\,\mathrm{d}y\mathrm{d}x$. (15%)

3. Find the Maclaurin Series for the following functions and determine their intervals of convergence. (10% each)
   (1) $f(x) = \ln(1+x)$.
   (2) $g(x) = \ln\left(\dfrac{1+x}{1-x}\right)$.

4. Find the following indefinite integrals. (5% each)
   (1) $\displaystyle\int \left(x^e + e^x + \dfrac{x}{e} + \dfrac{e}{x} + x + e\right)\mathrm{d}x.$
   (2) $\displaystyle\int \left(\sin(2x) + \cos(x+1) + \tan x\right)\mathrm{d}x.$
   (3) $\displaystyle\int \left(\dfrac{2}{1+x} - \dfrac{1}{1+x^2} + \dfrac{1}{\sqrt{1-x^2}}\right)\mathrm{d}x.$
   (4) $\displaystyle\int \left(\sec^2 x - \sec(2x)\tan(2x) + \sec x\right)\mathrm{d}x.$

5. Consider a unit sphere $S : x^2 + y^2 + z^2 = 1$. If we cut the sphere by the planes $x = -\dfrac{1}{3}$ and $x = \dfrac{1}{3}$, then we get three spherical shells, called A_1, A_2, and A_3. Find the surface areas of these spherical shells. (15%)

6. Let (15%)
$$f(x,y) = \begin{cases} \dfrac{x^3 - xy^2}{x^2 + y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0). \end{cases}$$
   (1) Find the partial derivative $f_x(0,0)$.
   (2) Find the gradient $\nabla f(1,2)$.