國立彰化師範大學110學年度碩士班招生考試試題

系所：數學系(選考丙)、統計資訊研究所(選考丙)　　科目：微積分

☆☆請在答案紙上作答☆☆　　

以下皆是計算題與證明題，必須書寫計算或證明過程。

1. **(10%)** Prove, by the $\varepsilon-\delta$ definition of limits, $\lim_{x\to a}\sqrt{x}=\sqrt{a}$ $(a>0)$.

2. **(20%)** Evaluate the following limits:

(a) $\lim_{x\to 0}\dfrac{x^2}{1-\cos x}$　(b) $\lim_{x\to\infty}\dfrac{(\ln x)^5}{\sqrt{x}}$　(c) $\lim_{x\to 0}(\cos x)^{1/x^2}$　(d) $\lim_{x\to\infty}(x-\sqrt{x})$

3. **(20%)** Find the following integrals:

(a) $\int e^{\sqrt{x}}\,dx$　(b) $\int_{-1}^{3}\dfrac{2x-4}{x^2-2x+5}\,dx$

4. **(30%)** Evaluate the following double integrals:

(a) $\iint_D xy\,dxdy$, where D is the region bounded by the line $y=x-1$ and the parabola $y^2=2x+6$.

(b) $\iint_R (x^2+y)\,dxdy$, where R is the region lying between the two circles $x^2+y^2=1$ and $x^2+y^2=5$.

(c) $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-(x^2+2xy+5y^2)}dxdy$

5. **(20%)** Test the series below for convergence.

(a) $\sum_{n=0}^{\infty}\dfrac{\sqrt{n+2}}{n^2-5n+1}$　(b) $\sum_{n=1}^{\infty}\dfrac{\ln(n+2)-\ln n}{\tan^{-1}(\frac{2}{n})}$　(c) $\sum_{n=1}^{\infty}\dfrac{n^{99}}{2^n}$　(d) $\sum_{n=2}^{\infty}\dfrac{1}{n\ln n}$