

# 國立彰化師範大學112學年度碩士班招生考試試題

系所：數學系(選考丙)、

科目：微積分

統計資訊研究所(選考丙)

☆☆請在答案紙上作答☆☆

共1頁，第1頁

1. (A) Suppose  $f$  is a function defined on an open interval containing  $c$ . (10%)

State the definition of  $\lim_{x \rightarrow c} f(x) = b$ .

(B) Apply the above definition to prove  $\lim_{x \rightarrow 2} \frac{1}{x} = \frac{1}{2}$

2. Find the following limits, if they exist: (10%)

(A)  $\lim_{x \rightarrow 1} \frac{x^{10}-1}{x^5-1}$       (B)  $\lim_{x \rightarrow 0^+} \frac{e^{-1/\sin x}}{x^2}$       (C)  $\lim_{x \rightarrow 0} (\cos 2x)^{\frac{1}{x^2}}$       (D)  $\lim_{x \rightarrow 0} \arctan\left(\frac{1}{x}\right)$

3. For the function  $f(x) = xe^{-x^2}$ , find: (10%)

- (A) all intervals where its graph is increasing or decreasing  
(B) all intervals where its graph concaves upward or downward  
(C) all local maxima and minima  
(D) all inflection points

4. Find the following integrals: (10%)

(A)  $\int_0^{\frac{1}{2}} x^2 \arcsin x \, dx$       (B)  $\int \frac{x^3+7x-13}{(x^2+4)(x-1)} dx.$

5. It is well known that  $\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$ . Find  $\int_{-\infty}^{\infty} (x-2)^2 e^{-4x^2} dx$ . (10%)

6. Find the sum of the series  $\sum_{n=1}^{\infty} \frac{n^2}{5^n}$  (10%)

7. Find the maximum and minimum values of  $f(x, y, z) = x^3 y^2 z$  on the ball (10%)

$$x^2 + y^2 + z^2 \leq 6.$$

8. Evaluate the double integral  $\iint_T (x-y)^2 dx dy$  over the triangle  $T$  with vertices (10%)  
(0,0), (4,0) and (2,2).

9. Evaluate the surface integral  $\iint_T z \, dS$  where  $T$  is the first-octant portion of the plane (10%)

$$x + y + z = 1.$$

10. Evaluate the triple integral  $\iiint_B (\sin(z) + xyz + z^2) \, dV$  where  $B$  is the ball given by (10%)

$$x^2 + y^2 + z^2 \leq 1.$$