國立彰化師範大學114學年度碩士班招生考試試題

系所： 數學系(選考丙)、
　　　　統計資訊研究所(選考丙)

科目： 微積分

☆☆請在答案紙上作答☆☆

共1頁，第1頁

You must state your reasons when answering or you will not get credit.

1. (10 %) Determine whether the limit exists or not. If the limit exists, find its value.

$$(a) \quad \lim_{x \to \infty} \left(x - \sqrt{x+1} \right) \qquad (b) \quad \lim_{x \to 0^+} (1 + \sin x)^{1/x}$$

2. (10 %) Define

$$f(x) = \begin{cases} x \ln |x| & \text{if } x \neq 0, \\ a & \text{if } x = 0. \end{cases}$$

   (a) Determine the value of a such that f is continuous at $x = 0$.

   (b) Determine whether f is differentiable at $x = 0$.

3. (10 %) Suppose that $y = f(x)$. Find $f'(1)$ if

$$e^y + \ln \left(\frac{1+x}{1+x^2} \right) = 1.$$

4. (15 %) Evaluate the following integral

$$(a) \quad \int_{-1}^2 \frac{1}{\sqrt[3]{x}} dx \qquad (b) \quad \int \cos (\ln x) dx \qquad (c) \quad \int_1^4 \frac{4x+1}{x^2 - 2x + 10} dx$$

5. (10 %) Let R be the region enclosed by $y = 2 - x^2, y = x$ and $y = 0$.

   (a) Evaluate the area of the region R.

   (b) Evaluate the volume of solid generated by rotating the region R about the x-axis.

6. (10 %) Given $f(x) = x - e \ln x, \ x > 0$.

   (a) Show that f is nonnegative for $x > 0$.

   (b) Use the result in part (a) to show that $e^\pi > \pi^e$.

7. (10 %) Suppose $f(x,y) = (x-1)y, \ D = \{(x,y) \in \mathbf{R}^2 \mid x^2 + y^2 \leq 1\}$. Find maximum and minimum of f on D

8. (10 %) It is known that $\int_{-\infty}^\infty e^{-x^2} dx = \sqrt{\pi}$. Find $\iint_{\mathbf{R}^2} e^{-(x^2 + xy + y^2)} dx dy$

9. (15 %) Find Maclaurin Series (Taylor Series about 0) for the following functions. What are their radius of convergence?

$$(a) \quad \arctan x \qquad (b) \quad (1 - x^2)^{-0.5} \qquad (c) \quad e^x \cos x, \ \text{upto the } x^4 \text{ term}$$