國立彰化師範大學105學年度碩士班招生考試試題

系所： 數學系　　　　　　　　　　　　　　科目： 線性代數

☆☆請在答案紙上作答☆☆　　　　　　　　　　

1. Let $T: \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation defined by

$T(x, y, z) = (x \quad 2y \quad z, \quad x \quad y \quad 2z, +$

Determine the rank of T and find the inverse if it exists. (20%)

2. Let the reduced echelon form of a matrix A be

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 - \\ 0 & 1 & 0 & 0 - & 1/2 & 0 \\ 0 & 0 & 1 & 1 & 1/2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 - \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

(1) Find a basis for solution space of $Ax = 0$. (5%)

(2) Determine A if the first, second, third, and sixth columns of A are

$$\begin{bmatrix} -2 \\ 0 \\ 0 \\ -1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -2 \\ -1 \\ -1 \end{bmatrix}, \begin{bmatrix} -5 \\ 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \text{ and } \begin{bmatrix} 4 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix},$$ respectively. (15%)

3. Find the determinant of the matrix $\begin{bmatrix} a & b & a+b \\ b & a+b & a \\ a+b & a & b \end{bmatrix}$. (10%)

4. Determine the characteristic polynomial, eigenvalues and eigenspaces of the matrix

$A = \begin{bmatrix} -5 & -12 & -20 \\ 2 & 5 & 10 \\ 0 & 0 & -1 \end{bmatrix}$. Find also an invertible matrix P that diagonalizes A. (20%)

5. Find an orthonormal basis for the subspace $\{(x, y, z, w) \mid x + y - z - w = 0\}$ in $\mathbb{R}^4$. (15%)

6. Suppose $\mathbf{v}_1 = \begin{bmatrix} p_{11} \\ p_{21} \end{bmatrix}$ and $\mathbf{v}_2 = \begin{bmatrix} p_{12} \\ p_{22} \end{bmatrix}$ are eigenvectors of 2×2 matrix A corresponding to eigenvalues λ_1 and λ_2, respectively. Show that $P^{-1}AP = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$, where $P = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix}$. (15%)