

# 國立彰化師範大學107學年度碩士班招生考試試題

系所： 數學系(選考甲)

科目： 線性代數

☆☆請在答案紙上作答☆☆

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1. Find all solutions of the linear system (20%)

$$3x_1 + x_2 + 4x_3 + 2x_4 = 2$$

$$2x_1 + x_2 + x_4 + x_5 = 3$$

$$x_1 + x_3 + 2x_5 = 4$$

2. Let  $A = \begin{bmatrix} 2 & 1 & -1 \\ 1 & 2 & 1 \\ 3 & 1 & 2 \end{bmatrix}$ . Express  $A$  as a product of elementary matrices. (20%)

3. In the inner-product space  $P_{0,1}$  of all polynomial function with real coefficients and domain  $0 \leq x \leq 1$ , and with inner product defined by

$$\langle f, g \rangle := \int_0^1 x f(x) g(x) dx.$$

Use the Gram-Schmidt process to find an orthogonal basis for the subspace

$$V = \text{sp}(1, x, x^2). \quad (20\%)$$

4. Find an orthogonal substitution that diagonalizes the quadratic form  $2xy + 2yz$ .

(20%)

5. Let  $v$  and  $w$  be vectors in  $\mathbb{R}^n$ . Show that  $|v \cdot w| \leq \|v\| \|w\|$ . (20%)