

# 國立彰化師範大學108學年度碩士班招生考試試題

系所：數學系(選考甲)、  
統計資訊研究所(選考甲)

科目：線性代數

☆☆請在答案紙上作答☆☆

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1. Find a basis for  $C^n$  over the field  $R$  of all real numbers. (20%)
2. Find an orthonormal basis for  $W = \text{sp}\{[1, 2, 3, 0], [1, 2, 0, 0], [1, 0, 0, 1]\}$  of  $R^4$ . (20%)
3. Find an orthogonal substitution that diagonalizes the quadratic form  $2xy + 2yz$ . (20%)
4. Prove that  $\det(A) = \prod_{1 \leq i < j \leq 5} (x_j - x_i)$ , where  $x_1, x_2, x_3, x_4, x_5 \in R$  and

$$A = \begin{bmatrix} 1 & x_1 & x_1^2 & x_1^3 & x_1^4 \\ 1 & x_2 & x_2^2 & x_2^3 & x_2^4 \\ 1 & x_3 & x_3^2 & x_3^3 & x_3^4 \\ 1 & x_4 & x_4^2 & x_4^3 & x_4^4 \\ 1 & x_5 & x_5^2 & x_5^3 & x_5^4 \end{bmatrix}. \quad (20\%)$$

5. Find an invertible matrix  $P$  and a diagonal matrix  $D$  such that  $P^{-1}BP = D$ , where

$$B = \begin{bmatrix} 2 & 0 & 0 \\ 1 & 2 & 1 \\ -1 & 0 & 1 \end{bmatrix}. \quad (20\%)$$