國立彰化師範大學111學年度碩士班招生考試試題

系所：數學系(選考甲)、統計資訊研究所(選考甲)　　　　科目：線性代數

☆☆請在答案紙上作答☆☆　　　　

1. (20%) Find the solution set of the system of linear equations

$$\begin{cases} x_1 + 2x_2 - x_3 + 3x_4 = 2 \\ 2x_1 + 4x_2 - x_3 + 6x_4 = 2. \\ x_1 + 3x_2 - x_3 + 5x_4 = 5 \end{cases}$$

2. (20%) Find an orthogonal or unitary matrix *P* and a diagonal matrix *D* such that *P*AP=D*, where

$$A = \begin{pmatrix} 3 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 3 \end{pmatrix}.$$

3. (20%) Let V=$P_3(R)$ with the inner product $\langle f(x),\ g(x)\rangle = \int_0^1 f(t)\, g(t)\, dt.$ Apply the Gram-Schmidt process to the subset $S = \{1, \mathrm{x}, x^3\}$ of the inner product space V to obtain an orthogonal basis β for span(S).

4. (20%) In R^2, let L be the line *y*=4*x*. Find an expression for **T**(*x*, *y*), where T is the reflection of R^2 about L.

5. (20%) Let **A** and **B** be n × n matrices with real entries. The trace of **A** is defined by

$$\mathrm{tr(A)} = \sum_{i=1}^{n} A_{i\,i}.$$

Prove that tr(**A B**)=tr(**B A**).