

國立彰化師範大學112學年度碩士班招生考試試題

系所：數學系(選考甲)、

科目：線性代數

統計資訊研究所(選考甲)

☆☆請在答案紙上作答☆☆

共1頁，第1頁

1. (20%) Let $V = P_3(\mathbb{R})$ with the inner product $\langle f, g \rangle = \int_0^1 f(t) g(t) dt$.

Apply the Gram-Schmidt process to $S = \{1, x, x^3\}$ to find an orthogonal basis β for $\text{span}(S)$.

2. (20%) Find new coordinates x', y' so that $9x^2 + 24xy + 16y^2 - 20x + 15y = 0$ can be written as $\lambda_1(x')^2 + \lambda_2(y')^2 + d x' + e y' + f = 0$, where $\lambda_1, \lambda_2, d, e$, and f are scalars.

3. (20%) Let $T: P_2(\mathbb{R}) \rightarrow P_2(\mathbb{R})$ be defined by $T(f(x)) = f(x) + (x-1)f'(x)$.

Let $\beta = \{1, x, x^2\}$ be the standard ordered basis for $P_2(\mathbb{R})$.

(1) Find the matrix representation $[T]_\beta$ of T in the basis β .

(2) Is T diagonalizable? Why?

4. (20%) Let $\begin{pmatrix} 1 & 2 & 0 & -3 & 0 & -1 \\ 0 & 0 & 1 & -5 & 0 & 2 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ be the reduced row echelon form of A . Determine A if

the second, fourth, and sixth columns of A are

$$\begin{pmatrix} 1 \\ -2 \\ -1 \\ 3 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ 2 \\ -4 \end{pmatrix} \text{ and } \begin{pmatrix} 3 \\ -9 \\ 2 \\ 5 \end{pmatrix}, \text{ respectively.}$$

5. (20%) Let $V = \{(x_1, x_2, x_3, x_4, x_5) \in \mathbb{R}^5: x_1 - 2x_2 + 3x_3 - x_4 + 2x_5 = 0 \text{ and } 2x_1 + x_2 + x_3 - x_5 = 0\}$.

(1) Find a basis for V .

(2) Let $S = \{(1, 0, -1, 0, 1)\}$. Extend S to a basis for V .