國立彰化師範大學114學年度碩士班招生考試試題

系所：數學系(選考甲)、統計資訊研究所(選考甲)　　科目：線性代數

☆☆請在答案紙上作答☆☆　　

1. (20%) Let $V = \boldsymbol{R^4}$ and W = span{[1, 1, 0, 2], [2, 0, 1, 1]}. Find a basis for $W^{\perp}$.

2. (20%) Let $T : P_2(\boldsymbol{R}) \longrightarrow P_2(\boldsymbol{R})$ be defined by $T\big(p(x)\big) = p(x+1) + p'(x)$, where $\beta = (x^2, x, 1)$ and $\beta' = (x^2+1,\ x+1,\ 2)$ are ordered bases for $P_2(\boldsymbol{R})$.
   (a) Find the matrix representations $[T]_{\beta}$ and $[T]_{\beta'}$ of T relative to β and β', respectively.
   (b) Find an invertible matrix *C* such that $[T]_{\beta'} = C^{-1}[T]_{\beta}\ C$.

3. (20%) Use rotation and translation of axes to sketch the curve $3x^2 - 10xy + 3y^2 + 16\sqrt{2}x = 32$.

4. (20%) Let $A = \begin{bmatrix} 3 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 3 \end{bmatrix}$. Find an orthogonal matrix P such that $P^{-1}AP$ is a diagonal matrix.

5. (20%) Let $B = \begin{bmatrix} 2 & -1 & 1 \\ 1 & 2 & s \\ s & s & 2 \end{bmatrix}$. Find all the real numbers *s* such that *B* is a singular matrix.