

# 國立彰化師範大學113學年度碩士班招生考試試題

系所：統計資訊研究所(選考乙)

科目：統計學

☆☆請在答案紙上作答☆☆

共3頁，第1頁

- (15%) 關於事件  $A$ 、 $B$ 、 $C$ ，已知  $P(A \cap B) = 0.24$ ， $P(A \cap B/\bar{C}) = 0.2$ ， $P(C) = 0.6$ ， $P(B|C) = 0.3$ 。 $\bar{C}$  表  $C$  的補集。求  $P(A \cap B \cap C)$ 。
- (15%) 對某政策的支持度進行民意調查；在 100 份有效問卷中，有 60 份支持，40 份反對。令  $p_1$  為一般民眾對該政策之支持率， $p_2$  為不支持率，請檢定：支持率高於不支持率 ( $H_1: p_1 > p_2$ )，顯著水準設定為 0.05。
- 某巴士有 18 個座位，而巴士公司為多賺一些利潤，通常會超賣 2 張票，賭有些乘客不會準時出現。假設每位客人出現與否為獨立事件，且不出現機率為 0.1。
  - (10%) 今某班車賣了 20 張票 (超賣 2 張)，請問屆時有出現乘客有票無位的機率是多少？
  - (10%) 假設票價為 500 元；但若人出現而沒位子坐，公司得補償 2 倍票款。請問就長期獲利而言，超賣 1 張跟超賣 2 張，哪種策略較好？
- (5%) State the central limit theorem (CLT).
  - (10%) Let a random variable  $X$  with probability density function (pdf)  $f_X(x) = cx^{99}e^{-\frac{x}{5}}$ ,  $x \geq 0$ ; zero otherwise, where  $c$  is a constant. Please use CLT to approximate  $P(450 < X < 600)$ .
- Let  $X_1, X_2, \dots, X_n$  be independent and identically distributed uniform observations on the interval  $(\theta, \theta + 1)$ ,  $-\infty < \theta < \infty$ . Let  $X_{(1)} < \dots < X_{(n)}$  be the order statistics from the sample.
  - (5%) Find the maximum likelihood estimator(s) of  $\theta$ .
  - (5%) Find the sufficient statistic for  $\theta$ .
  - (5%) Please find a moment estimator for  $\theta$ . Is this moment estimator an unbiased estimator for  $\theta$ ? explain it.
  - (10%) The joint pdf of  $X_{(1)}$  and  $X_{(n)}$  is
$$f(x_{(1)}, x_{(n)} | \theta) = \begin{cases} n(n-1)(x_{(n)} - x_{(1)})^{n-2}, & \theta < x_{(1)} < x_{(n)} < \theta + 1 \\ 0, & \text{otherwise.} \end{cases}$$
Let  $R = X_{(n)} - X_{(1)}$  and  $M = (X_{(1)} + X_{(n)})/2$ , please find the joint pdf of  $R$  and  $M$ .
  - (10%) Find the marginal pdf of  $R$ . What is the distribution and corresponding parameters of  $R$ ?

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共3頁，第2頁

Standard Normal Table

$$P(Z \leq z) = \Phi(z) = \int_{-\infty}^z \frac{1}{\sqrt{2\pi}} e^{-w^2/2} dw$$

$$[\Phi(-z) = 1 - \Phi(z)]$$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7703	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
$\alpha$	0.400	0.300	0.200	0.100	0.050	0.025	0.010	0.005	0.001	
$Z_{\alpha}$	0.253	0.524	0.842	1.282	1.645	1.960	2.326	2.576	3.090	
$Z_{\alpha/2}$	0.842	1.036	1.282	1.645	1.960	2.240	2.576	2.807	3.291	

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共3頁，第3頁

Numbers in the table represent  $P(X=x)$  for a binomial distribution with  $n$  trials and probability of success  $p$ .  $\binom{n}{x} p^x (1-p)^{n-x}$

n	x	p																		
		.01	.05	.10	.15	.20	.25	.30	.35	.40	.45	.50	.55	.60	.65	.70	.75	.80	.85	.90
19	0	.8262	.3774	.1351	.0456	.0144	.0042	.0011	.0003	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	1	.1586	.3774	.2852	.1529	.0685	.0268	.0093	.0029	.0008	.0002	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	2	.0144	.1787	.2852	.2428	.1540	.0803	.0358	.0138	.0046	.0013	.0003	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	3	.0008	.0533	.1796	.2428	.2182	.1517	.0869	.0422	.0175	.0062	.0018	.0005	.0001	.0000	.0000	.0000	.0000	.0000	.0000
	4	.0000	.0112	.0798	.1714	.2182	.2023	.1491	.0909	.0467	.0203	.0074	.0022	.0005	.0001	.0000	.0000	.0000	.0000	.0000
	5	.0000	.0018	.0266	.0907	.1636	.2023	.1916	.1468	.0933	.0497	.0222	.0082	.0024	.0006	.0001	.0000	.0000	.0000	.0000
	6	.0000	.0002	.0069	.0374	.0955	.1574	.1916	.1844	.1451	.0949	.0518	.0233	.0085	.0024	.0005	.0001	.0000	.0000	.0000
	7	.0000	.0000	.0014	.0122	.0443	.0974	.1525	.1844	.1797	.1443	.0961	.0529	.0237	.0083	.0022	.0004	.0000	.0000	.0000
	8	.0000	.0000	.0002	.0032	.0166	.0487	.0981	.1489	.1797	.1771	.1442	.0970	.0532	.0233	.0077	.0018	.0003	.0000	.0000
	9	.0000	.0000	.0000	.0007	.0051	.0198	.0514	.0980	.1464	.1771	.1762	.1489	.0976	.0528	.0220	.0066	.0013	.0001	.0000
	10	.0000	.0000	.0000	.0001	.0013	.0066	.0220	.0528	.0976	.1449	.1762	.1771	.1464	.0980	.0514	.0198	.0051	.0007	.0000
	11	.0000	.0000	.0000	.0000	.0003	.0018	.0077	.0233	.0532	.0970	.1442	.1771	.1797	.1489	.0981	.0487	.0166	.0032	.0002
	12	.0000	.0000	.0000	.0000	.0000	.0004	.0022	.0083	.0237	.0529	.0961	.1443	.1797	.1844	.1525	.0974	.0443	.0122	.0014
	13	.0000	.0000	.0000	.0000	.0000	.0001	.0005	.0024	.0085	.0233	.0518	.0949	.1451	.1844	.1916	.1574	.0955	.0374	.0069
	14	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0006	.0024	.0082	.0222	.0497	.0933	.1468	.1916	.2023	.1636	.0907	.0266
	15	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0005	.0022	.0074	.0203	.0467	.0909	.1491	.2023	.2182	.1714	.0798
	16	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0005	.0018	.0062	.0175	.0422	.0869	.1517	.2182	.2428	.1796
	17	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0003	.0013	.0046	.0138	.0358	.0803	.1540	.2428	.2852
	18	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0002	.0008	.0029	.0093	.0268	.0685	.1529	.2852
	19	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0003	.0011	.0042	.0144	.0456	.1351
20	0	.8179	.3585	.1216	.0388	.0115	.0032	.0008	.0002	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	1	.1652	.3774	.2702	.1368	.0576	.0211	.0068	.0020	.0005	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	2	.0159	.1887	.2852	.2293	.1369	.0669	.0278	.0100	.0031	.0008	.0002	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	3	.0010	.0596	.1901	.2428	.2054	.1339	.0716	.0323	.0123	.0040	.0011	.0002	.0000	.0000	.0000	.0000	.0000	.0000	.0000
	4	.0000	.0133	.0898	.1821	.2182	.1897	.1304	.0738	.0350	.0139	.0046	.0013	.0003	.0000	.0000	.0000	.0000	.0000	.0000
	5	.0000	.0022	.0319	.1028	.1746	.2023	.1789	.1272	.0746	.0365	.0148	.0049	.0013	.0003	.0000	.0000	.0000	.0000	.0000
	6	.0000	.0003	.0089	.0454	.1091	.1686	.1916	.1712	.1244	.0746	.0370	.0150	.0049	.0012	.0002	.0000	.0000	.0000	.0000
	7	.0000	.0000	.0020	.0160	.0545	.1124	.1643	.1844	.1659	.1221	.0739	.0366	.0146	.0045	.0010	.0002	.0000	.0000	.0000
	8	.0000	.0000	.0004	.0046	.0222	.0609	.1144	.1614	.1797	.1623	.1201	.0727	.0355	.0136	.0039	.0008	.0001	.0000	.0000
	9	.0000	.0000	.0001	.0011	.0074	.0271	.0654	.1158	.1597	.1771	.1602	.1185	.0710	.0336	.0120	.0030	.0005	.0000	.0000
	10	.0000	.0000	.0000	.0002	.0020	.0099	.0308	.0686	.1171	.1593	.1762	.1593	.1171	.0686	.0308	.0099	.0020	.0002	.0000
	11	.0000	.0000	.0000	.0000	.0005	.0030	.0120	.0336	.0710	.1185	.1602	.1771	.1597	.1158	.0654	.0271	.0074	.0011	.0001
	12	.0000	.0000	.0000	.0000	.0001	.0008	.0039	.0136	.0355	.0727	.1201	.1623	.1797	.1614	.1144	.0609	.0222	.0046	.0004
	13	.0000	.0000	.0000	.0000	.0000	.0002	.0010	.0045	.0146	.0366	.0739	.1221	.1659	.1844	.1643	.1124	.0545	.0160	.0020
	14	.0000	.0000	.0000	.0000	.0000	.0000	.0002	.0012	.0049	.0150	.0370	.0746	.1244	.1712	.1916	.1686	.1091	.0454	.0089
	15	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0003	.0013	.0049	.0148	.0365	.0746	.1272	.1789	.2023	.1746	.1028	.0319
	16	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0003	.0013	.0046	.0139	.0350	.0738	.1304	.1897	.2182	.1821	.0898
	17	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0002	.0011	.0040	.0123	.0323	.0716	.1339	.2054	.2428	.1901
	18	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0002	.0008	.0031	.0100	.0278	.0669	.1369	.2293	.2852
	19	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0005	.0020	.0068	.0211	.0576	.1368	.2702
	20	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0002	.0008	.0032	.0115	.0388	.1216